Year 13 Further Maths

$$\text{Let } f(x) = \arcsin x$$

Find the exact mean value of $f(x)$ over the interval $\left[0, \frac{1}{2}\right]$

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The mean value of $f(x)$ over the interval $\left[0, \frac{1}{2}\right]$ is given by the formula

$$\frac{1}{\frac{1}{2} - 0} \int_0^{\frac{1}{2}} \arcsin(x)\, dx$$

Therefore the mean value will be

$$2 \int_0^{\frac{1}{2}} 1 \times \arcsin(x)\, dx$$

Which we integrate by parts with

$$u = \arcsin x \qquad v' = 1$$

$$u = \frac{1}{\sqrt{1 - x^2}} \qquad v = x$$

So the mean value will be

$$2 \times \left(\left[x \arcsin x \right]_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}} \frac{x}{\sqrt{1-x^2}}\, dx \right)$$

$$= 2 \times \left(\left[x \arcsin x \right]_0^{\frac{1}{2}} + \frac{1}{2} \int_0^{\frac{1}{2}} \frac{-2x}{\sqrt{1-x^2}}\, dx \right)$$

This we can integrate using the reverse chain rule

$$= 2 \times \left(\left[x \arcsin x + \sqrt{1-x^2} \right]_0^{\frac{1}{2}} \right)$$

$$= 2\left(\frac{1}{2} \arcsin \frac{1}{2} + \sqrt{1 - \frac{1}{4}} \right) - 2\left(0 + \sqrt{1}\right)$$

$$= \frac{\pi}{6} + 2\sqrt{\frac{3}{4}} - 2$$

$$= \frac{\pi}{6} + \sqrt{3} - 2$$