Year 13 Maths

$$f(x) = |ax + b| - c$$

$$a, b, c > 0$$

Find, in terms of $a, b,$ and c :

- The coordinates of the intersections of $f(x)$ with the axes
- The domain of $f(x)$
- The range of $f(x)$

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x-axis coordinates:

The function crosses the x-axis when $y = 0$.

So:

$$|ax + b| - c = 0$$
$$\implies |ax + b| = c$$

So, either

$$ax + b = c$$

or

$$-(ax + b) = c$$

depending on whether the function is positive or negative inside the modulus.

Therefore

$$ax = c - b$$
$$\implies x = \frac{c - b}{a}$$

for the positive case.

And

$$ax + b = -c$$
$$\implies ax = -b - c$$
$$\implies x = \frac{-b - c}{a} = -\frac{b + c}{a}$$

for the negative case.

So the coordinates where the function crosses the x-axis are

$$\left(-\frac{b + c}{a}, 0\right) \text{ and } \left(\frac{c - b}{a}, 0\right)$$

y-axis coordinates:
The function crosses the y-axis when $x = 0$.

So:

$$y = |a \times 0 + b| - c$$
$$\implies y = |b| - c$$
$$\implies y = b - c$$

as $b > 0$.

So the coordinates where the function crosses the y-axis is

$$(0, b - c)$$

There are no restrictions on the values of x that we can put into the function so the domain is $x \in \mathbb{R}$.

The modulus function is always greater than or equal to zero, therefore if we subtract c from this, the range of the function will be $f(x) \geq -c$.