Further Maths Core Pure 2 Practice Paper 2025

1. Let $f(x) = \cos x$. The mean value of $f(x)$ over the interval $[0, a]$ is 0.

(a) Write down a possible value of a.

(1)

(b) Let $g(x) = \arccos x$.

Find the exact mean value of $g(x)$ over the interval $[0, \frac{1}{2}]$.

(4)

(Total for Question 1 is 5 marks)

2. (a) Prove by induction that

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}^n = \begin{pmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{pmatrix}$$

for all integers $n \geq 1$.

(5)

(b) (i) Explain why $5^a - 15$ is always a multiple of 10 for all integer values of $a \geq 1$.

(1)

(ii) Let $f(k) = 4^{2k} + 5^k - 1$.

By considering $f(k+1) - 16f(k)$, or otherwise, prove by induction that $f(k)$ is always divisible by 10 for all integer values $k \geq 1$.

(4)

(Total for Question 2 is 10 marks)

3. The line l_1 has equation $\mathbf{r}_1 = \begin{pmatrix} 1 \\ a \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 2 \\ b \end{pmatrix}$.

(a) The point $A(7, 2, -11)$ lies on the line l_1.

Show that $a = 6$ and $b = 5$.

(3)

(b) Find the shortest distance from the point $B(1, 2, -7)$ to the line l_1. Give your answer in the form $\sqrt{k}$, where k is a constant to be determined.

(4)

(Total for Question 3 is 7 marks)

4. The transformation represented by the 2×2 matrix A maps the point $(1, -3)$ to $(-9, 17)$ and has an invariant point at $(3, 1)$.

(a) Find the matrix A.

(3)

(b) Determine all invariant lines of A.

(5)

(c) Find the image of the straight line $y = 2x + 1$ under the transformation A. Give your answer in the form $ay + bx + c = 0$, where $a, b,$ and c are integers to be determined.

(4)

(Total for Question 4 is 12 marks)

5. You are given that

$$\sum_{k=3}^{18} f(k) = 728$$

for some function $f(k)$.

(a) Given that $f(k) = ak + b$, where a and b are constants, show that $21a + 2b = 91$.

(3)

(b) Given further that

$$\sum_{k=a}^{10} f(k) = 183$$

find the values of a and b.

(5)

(Total for Question 5 is 8 marks)

6. (a) Prove that

$$\tan 5\theta \equiv \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}.$$

(5)

(b) Using part (a), or otherwise, prove that

$$\tan\frac{\pi}{5} = \sqrt{5 - 2\sqrt{5}}.$$

(4)

(c) Prove also that

$$\sec^2\frac{\pi}{5} + \sec^2\frac{2\pi}{5} = 12.$$

(3)

(Total for Question 6 is 12 marks)

7. The plane Π_1 has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ and the plane Π_2 has equation $2x - 4y - z = 5$.

(a) Show that Π_1 and Π_2 are parallel planes.

(4)

(b) Find the vertical distance between the planes Π_1 and Π_2.

(4)

(c) A third plane Π_3 has equation $\mathbf{r} \cdot \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = 4$.

Find the acute angle between Π_1 and Π_3.

(3)

(Total for Question 7 is 11 marks)

8. The polar curve C_1 has equation $r = 2\sec\left(\theta + \frac{\pi}{3}\right)$.

(a) Prove that C_1 is a straight line and find its equation.

(3)

(b) The polar curve C_2 has equation $r = 3 + \cos^2\theta$.

(i) Show that the Cartesian equation of C_2 is $(x^2 + y^2)^{\frac{3}{2}} = 4x^2 + 3y^2$.

(3)

(ii) Prove that the gradient function of C_2 can be written in the form

$$\frac{dy}{dx} = \frac{-\cot\theta(a\cos^2\theta + 1)}{b(\cos^2\theta + 1)}$$

where a and b are integers to be determined.

(4)

(Total for Question 8 is 10 marks)