

Hypothesis Testing

Section A

1. A statistical test is carried out as below:

$$H_0 : p = \frac{1}{6} \quad H_1 : p \leq \frac{1}{6}$$

Under H_0 , $X \sim B(26, \frac{1}{6})$. Significance level = 10%. Observed value = 1.

$P(X \leq 1) = 0.956 > 0.1$, therefore there is insufficient evidence to reject H_0 .

Find all the mistakes in the hypothesis test above.

2. A dice is thrown 50 times and lands on 6 five times. Test, at the 5% significance level, whether there is evidence that the dice is biased or not.
3. It was reported that the probability that a chess player will win a chess game is 0.8. In an effort to improve their win percentage the player reads a book from a grandmaster.

After reading the book, the chess player plays 20 games and wins 18. Test, at the 5% significance level, whether the chess player has improved their win percentage.

4. At a garden centre, the manager states that 30% of customers buy an extra item at the tills on the way out. A manager from another branch suggests this value is incorrect and carries out a test. She takes a sample of 40 customers.

(a) Find the critical region of a test which tests the manager's belief at the 2.5% level.

In the sample 5 customers purchase an extra item at the tills on the way out.

(b) Determine if there is sufficient evidence to support the manager's belief.

5. A physiotherapist reports that 75% of their patients report no pain within a 3 month period of starting therapy. A colleague believes that a higher proportion of his patients report no pain within 3 months.

A random sample of 50 of his patients that have finished therapy is taken and 40 reported having no pain within 3 months.

(a) Test, at the 5% significance level, the colleague's belief.

(b) Find the critical region for this test.

(c) State the actual significance level of this test.

Section B

1. A factory produces glass jars and finds that $p\%$ of jars, where p is an integer, have cracked by the end of the assembly line. A component of the assembly line is replaced and the supervisor believes this will lower the percentage of jars that will have cracks.

A random sample of 40 jars is taken after the component has been replaced. A test is carried out at the 2.5% significance level. At the 2.5% significance level, the critical region is found to be $X = 0$.

Show that $p = 9$.